

DETERMINATION AND CALCULATION OF THE
OPTIMUM SURFACE OF A CONTINUOUSLY
ACTING VACUUM-SUBLIMATION CONDENSER

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A method of calculating the optimum shape of the working surface of cooled condenser elements used in vacuum-sublimation installations is presented; an engineer's computing system for the design of continuously acting condensers of this kind is also given.

In perfecting the design of installations for the sublimation drying of food products, biological specimens, and other materials of low heat resistance, a number of problems arise in connection with the mass transfer and hydrodynamics of the vapor-gas mixture between the source of vapor evolution (sublimator) and the mass sink (condenser) [1]. The solution of these problems (with due allowance for the actual processes taking place on the exchange surface) enables us to determine the best shapes for the sublimator and condenser working surfaces, as well as their mutual disposition and orientation. The main requirements to be satisfied are: the organization of a continuous outflow of vapor from the sublimating source, with a minimum external resistance; uniform solidification of the condensate; the removal of uncondensed gases; maintenance of the optimum thermodynamic parameters of the process (pressure, ambient temperature, surface temperature).

One of the most complicated processes associated with any determination of the shape of the condenser surface is that of obtaining the flux-density distribution function of the vapor. Earlier we developed a method [2] enabling us to estimate the values of the effective expansion angle of the flow of water vapor emitted by a plane sublimating source, i.e., the values of the solid angle $2\varphi_{\text{eff}}$ in which the vapor flow was dense enough to form and grow solid condensate crystals (the temperature characterizing the operation of the condenser in the vacuum installation being 230–250°K).

Experimental investigations were carried out for the molecular-viscous, transitional, and viscous states of the vapor-gas mixture ($10 < p_c < 10^3 \text{ N/m}^2$). The model of sublimating material employed was water occluded in a porous metal, liquid-ice-vapor transformations taking place in the pores under the influence of the thermal flux (heat flow) created by a set of plane heaters.

Whereas in a high vacuum ($\text{Kn} \gg 1$) the molecular beam from the sublimating plane source weakened in accordance with the laws of ray optics, on increasing the ambient pressure we obtained a more substantial increase in the size of the solid condensate spot in a plane condenser (the spot was limited by the solid angle $2\varphi_{\text{eff}}$). Here

$$\cos \varphi_{\text{eff}} = \exp [k (p_c/p_0)^n]. \quad (1)$$

For the free-molecular state ($\text{Kn} \gg 1$) $k = 0$, $n = 0$; for the transitional state ($\text{Kn} \sim 1$) $k = -1$; $n = 1$; for the viscous state ($\text{Kn} < 1$) $k = -0.42$; $n = 0.5$.

Use of the shadow method of visualizing the hydrodynamic pattern of the flow around surfaces of various configurations revealed the existence of vapor-gas sheaths close to the condensation surfaces; these sheaths increased the resistance opposing the diffusion of vapor molecules and their complexes toward the surface, and (in the case of a cylinder) created conditions conducive toward the uniform distribution of the condensate over the cooled surface. A more uniform thickness (and resistance) of the boundary

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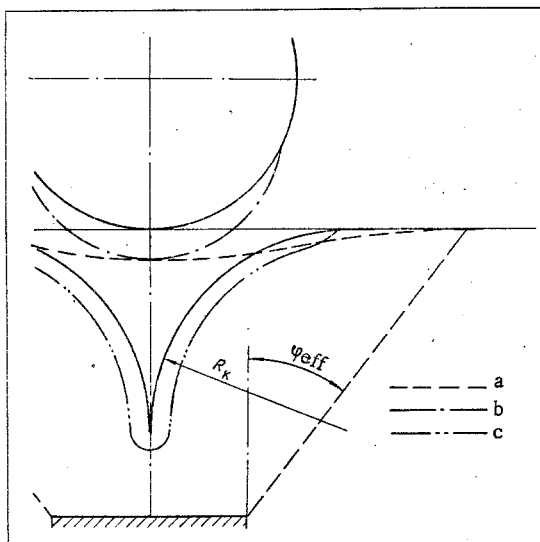


Fig. 1

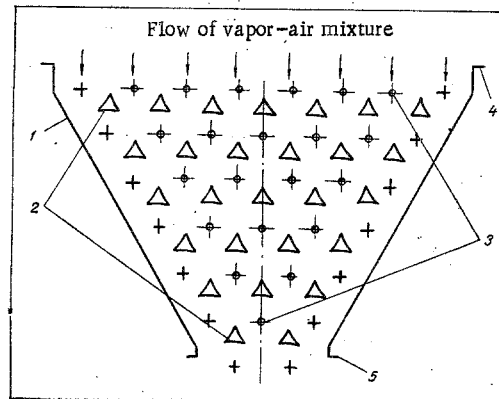


Fig. 2

Fig. 1. Distribution of the solid condensate of water vapor on cooled surfaces of various configurations: a) plane; b) cylinder; c) surface of optimum shape.

Fig. 2. Arrangement of the continuously acting vacuum-sublimation condenser: 1) body; 2) cooled elements; 3) scraper axes; 4, 5) flanges. The faces of elements 2 are arcs of common circles.

sheath close to the wall is obtained in the case of flow around an infinite flat plate set at right angles to the vapor flow. For this case we obtained a distribution of the solid condensate $h = f(R)$ corresponding to the following density distribution of the vapor flow from the center to the periphery:

$$I = I_s / \beta^2 \exp [(R/R_s \beta)^2 + k (p_c / p_0)^n]. \quad (2)$$

The investigations showed that plane and cylindrical condenser surfaces were not the best shapes as regards uniformity of condensate solidification. Earlier we proposed [3] a surface shape for the cooled elements such as would satisfy the requirement of uniformity, and which could be calculated very simply, the characteristic dimension of the surface (radius of curvature R_K) being determined in terms of the specified size of the source and the thermodynamic parameters of the ambient.

Figure 1 gives a normal cross section of the distribution of solid condensate on plane (a), cylindrical (b), and optimum-shaped (c) cooled surfaces (surrounding pressure 133 N/m^2 , condenser temperature 240°K , distance to vapor source 0.1 m). The most uniform distribution of the condensate was obtained in case (c) for a mean thickness of the solidified layer up to 0.01 m .

The model tests showed that a correctly chosen working surface of the condenser ensured uniform freezing of the solid condensate over a very long period of time.

Figure 2 illustrates the arrangement of our continuously acting vacuum-sublimation condenser. In the evacuated body of the condenser 1 are the cooled elements 2, made in the form of hollow trihedral prisms with concave faces having identical radii of curvature, and between the cooled elements 2 are scrapers 3 rotating around the axis for periodically cleaning the working surfaces of 2 from the solid condensate. Each scraper cleans the adjacent surfaces of three neighboring elements 2 at the same time in one pass. The variable-section body has walls contracting toward the outlet and two flanges: flange 4 for fixing to the sublimation chamber and flange 5 for connecting to the discharging caisson bunker and the system evacuating the uncondensed gases [5].

A thermal calculation may be executed for such a condenser if we know its rate of delivery, the radius of curvature R_K , the mean thermophysical characteristics of the solid condensate, and the thermodynamic parameters of the vapor-gas mixture.

Figure 3 shows the cross section of a cooling element of optimum shape in the form of a hollow prism with concave faces having a radius of curvature R_K . Uniform layers of solid condensate with a

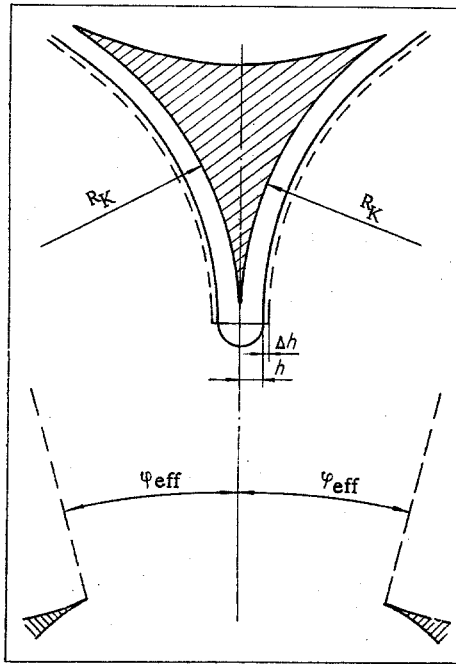


Fig. 3

Fig. 3. Optimum shape of the working surface of a cooled element of the condenser (to aid in calculating the freezing time).

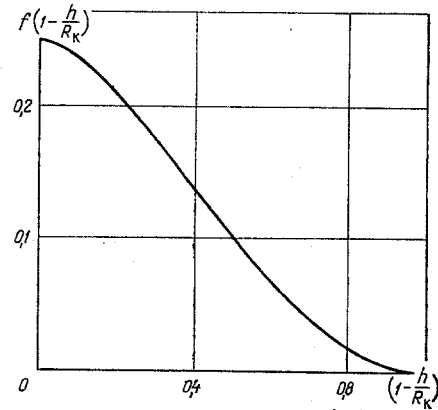


Fig. 4

Fig. 4. Characteristics of the thickness of the solid condensate layer freezing on the working surface.

thickness h freeze on two of the working faces. By virtue of the symmetry of the flow and the cooled element we need only consider one of the faces.

On increasing the thickness of the layer of condensate by an amount Δh a quantity ΔG of condensate freezes on the surface of the element:

$$\Delta G = \frac{\pi}{3} \rho L \Delta h (R_K - h - \Delta h/2) \quad (3)$$

(subsequently we shall everywhere neglect the term $\Delta h/2$).

Assuming that at the phase interface the thermal resistance of the phase transition is equal to zero, we may determine the amount of heat which has to be taken away from the moving boundary of the phase transition:

$$\Delta Q' = \Delta G r.$$

Part of the heat $\Delta Q''$ is carried to the phase-transition surface on cooling the vapor-gas mixture to the temperature of this surface T_s :

$$\Delta Q'' = \frac{1}{\epsilon} c_p (T_c - T_s) \Delta G.$$

The total amount of heat which has to be carried away when a layer of thickness Δh freezes is

$$\Delta Q = \frac{\pi}{3} \rho L \left[\frac{1}{\epsilon} c_p (T_c - T_s) + r \right] (R_K - h) \Delta h. \quad (4)$$

In the quasi-steady-state approximation the heat-transfer intensity will be determined by the thermal conductivity of the freezing layer. In our present case the heat-conduction equation for an element of the cylindrical surface may be expressed [4] as follows:

$$q_T = \frac{\pi L \lambda (T_s - T_K)}{3 \ln [R_K / (R_K - h)]}. \quad (5)$$

The time required for the phase interface to move by Δh for a heat-outflow intensity of q_T is given by $\Delta\tau = \Delta Q/q_T$:

$$\Delta\tau = \frac{\rho}{\lambda(T_s - T_R)} \left[\frac{1}{\varepsilon} c_p (T_c - T_s) + r \right] (R_K - h) \ln \left(\frac{R_K}{R_K - h} \right) \Delta h. \quad (6)$$

Denoting

$$K = \frac{\rho R_K^2}{\lambda(T_s - T_R)} \left[\frac{1}{\varepsilon} c_p (T_c - T_s) + r \right] \quad (7)$$

and integrating both sides of Eq. (6), we obtain the time required for the thickness of the solid condensate to increase from 0 to h :

$$\int dt = K \int_0^h \left(1 - \frac{h}{R_K} \right) \ln \left(1 - \frac{h}{R_K} \right) \left(-\frac{dh}{R_K} \right),$$

where

$$\tau = K \frac{1}{4} \left[2 \left(1 - \frac{h}{R_K} \right)^2 \ln \left(1 - \frac{h}{R_K} \right) - \left(1 - \frac{h}{R_K} \right)^2 + 1 \right] = K f \left(1 - \frac{h}{R_K} \right), \quad (8)$$

where K depends on the thermodynamic parameters of the process while the function $f(1-h/R_K)$ illustrated in Fig. 4 characterizes the thickness of the condensate layer.

For the case of flat cooled surfaces

$$\Delta\tau = \frac{\left[\frac{1}{\varepsilon} c_p (T_c - T_s) + r \right] \rho h \Delta h}{\lambda(T_s - T_R)}.$$

Integrating, we obtain an expression for h on the plane:

$$h = \sqrt{\frac{2\lambda(T_s - T_R)\tau}{\left[\frac{1}{\varepsilon} c_p (T_c - T_s) + r \right] \rho}}. \quad (9)$$

Thus in the quasi-steady-state approximation, which is perfectly satisfactory for engineer's calculations (for $0 < h/R_K < 0.4$), we obtain a reasonably simple computing system.

NOTATION

T_c, p_c, c_p	are the temperature, pressure, and specific heat of the vapor-gas mixture, respectively;
ε	is the weight proportion of vapor in the mixture;
p_0	is the water vapor pressure at the "triple point";
φ_{eff}	is the effective expansion angle of the vapor flow;
$I_{\text{so}}, R_{\text{so}}$	are the intensity and radius of the sublimation source;
T_s, T_K	are the temperatures of the condensation surface and of the condenser;
$\rho, \lambda, h, G, \beta$	are the density, thermal conductivity, and thickness of the layer, weight and profile characteristic of the solid condensate;
r	is the heat of desublimation;
R_K, L	are the radius of curvature and length of the cooled condenser element;
k, n	are the empirical coefficients.

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